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"MOSCOW POLYTECHNIC UNIVERSITY"

(MOSCOW POLYTECHNIC UNIVERSITY)

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Faculty of Economics and Management

APPROVED BY

FEU Dean

_____ /A.V. Nazarenko/

«27» February 2025

WORKING PROGRAM OF THE DISCIPLINE

«Linear algebra»

Direction of training/specialty

38.03.01 Economy

Profile/specialization

International business and foreign economic

Qualification

bachelor

Forms of study

full-time

Moscow, 2025

Разработчик:

Канд. физ.-мат. наук, доцент _____ /Е.А. Коган/

И.о. зав. кафедрой «Математика»,
канд. физ.-мат. наук _____ /Н.В. Васильева/

Agreed:

Head of Department"Economics and Organization"

PhD in Economics, Associate Professor _____ /A.V. Tenishchev/

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1. Objectives, tasks and planned learning outcomes discipline

To the main ones*goals*mastering the discipline "Linear Algebra" should include:
- developing a general mathematical culture in students;
- acquisition by students of a wide range of mathematical knowledge, skills and abilities kov;
- development of students' ability for inductive and deductive thinking along with the development of mathematical intuition;
- the ability of students to develop skills for independent study of educational and scientific literature containing mathematical information and results;
- preparation of students for activities in accordance with the qualification characteristics bachelor's degree in this field, including the development of skills to use mastered mathematical methods in professional activities.
- training of highly qualified personnel in demand in the digital environment turbulence and high technological risks of the modern digital economy.

To the main ones*tasks*mastering the discipline "Linear Algebra" should include:
- students master the basic concepts and methods that form general mathematical technical training necessary for the successful solution of applied problems;
- the formation of the required set of competencies in the student that correspond to his direction of training and ensuring its competitiveness in the labor market.

Training in the discipline "Linear Algebra" is aimed at developing the following competencies in students in accordance with Federal State Educational Standard 38.03.01 (bachelor's degree level) in the field of training 38.03.01 Economics, approved by order of the Ministry of Education and Science of Russia dated 12.08.2020 No. 954 (as amended and supplemented on November 26, 2020):

Code and name of competencies	**Indicators of achievement of comp-Tensions**
UK-1. Capable of searching, critically analyzing and synthesizing information, applying a systematic approach to solving assigned tasks	IUK-1.1. Analyzes the task, identifying its basic components IUK-1.2. Searches, critically evaluates, generalizes, systematizes and ranks the information required to solve the assigned task IUK-1.3. Considers and proposes rational options for solving a given problem using a systems approach, critically evaluates their advantages and disadvantages

2. The place of the discipline in the structure of the educational program

The discipline belongs to the compulsory part of block B1: Module “Mathematical and natural science disciplines”.
Its study ensures the study of disciplines *In the mandatory part*:
- mathematical analysis;

- econometrics;
- economic statistics;
- risk assessment and analysis.

In the part formed by the participants of educational relations:

- financial analysis;
- business planning.

3. Structure and content of the discipline

The total workload of the course is 3 credit units - 108 hours.

3.1. Types of academic work and labor intensity

3.1.1. Full-time education

p/p	Type of academic work	Quantity hours	Semester 1
	Classroom lessons	48	48
	Including:		
1.	Lectures	16	16
2.	Seminars/practical classes	32	32
3.	Laboratory exercises	-	-
	Independent work	60	60
	Interim assessment		
	exam	E	E
	Total	108	108

3.1.2. Part-time and part-time education

p/p	Type of academic work	Quantity hours	Semester 1
	Classroom lessons	28	28
	Including:		
. 1	Lectures	8	8
.2	Seminars/practical classes	20	20
	Laboratory exercises	-	-

	Independent work	80	80
	Interim assessment		
	exam	E	E
	Total	108	108

3.2. Thematic plan for studying the discipline

Placed in Appendix 1 to the work program.

3.3. Contents of discipline sections

Introduction

Subject, objectives and content of the discipline. Main stages of development of the discipline. Structure of the course, its place and role in the preparation of a bachelor, connection with other disciplines.

Section 1. Elements of Linear Algebra

Topic 1. Matrices and determinants.

The concept of a matrix. Types of matrices. Actions on matrices. Operations on matrices and their properties. Determinants, their properties and calculations. The concepts of minor and algebraic complement. Decomposition of a determinant into elements of a row or column. Calculation of determinants of different orders.

Inverse matrix and algorithm for its calculation. Elementary matrix transformations. Reduction of a matrix to diagonal or trapezoidal form. Matrix form of recording a system of linear algebraic equations. Matrix rank.

Topic 2. Solving systems of linear algebraic equations.

Systems of linear algebraic equations, basic concepts. Solution of systems of linear equations by Cramer's method, inverse matrix method, Gauss method. Kronecker-Capelli theorem. Solution of arbitrary systems of linear equations by Gauss method. Solution of homogeneous systems of linear equations. Criterion of existence of non-trivial solutions.

Section 2. Elements of vector algebra

Topic 1. Linear operations on vectors, their properties. Linear combinations of vectors. tori. Linear dependence and independence of vectors. Basis of a system of vectors. Uniqueness of decomposition of a vector with respect to a basis. Orthonormal basis.

Topic 2. Scalar, vector and mixed products of vectors, their properties. Conditions orthogonality, collinearity, coplanarity of vectors. Calculation of scalar, vector and mixed products of vectors specified by coordinates in an orthonormal basis.

Topic 3. Linear spaces. The concept of the basis of the space of geometric vectors tori. Transformation of the matrix of a linear operator when passing from basis to basis. Eigenvalues and eigenvectors of the matrix.

Section 3. Complex numbers and polynomials

The concept of a complex number. Forms of writing complex numbers. Operations on complex numbers. Moivre's formula. Extracting the positive n-th root of a complex number. Factorization of a polynomial. The fundamental theorem of algebra.

Section 4. Analytical Geometry

Coordinate systems. Cartesian rectangular coordinates on the plane and in space. Polar coordinate system.

Topic 1. Straight lines and planes

Different types of plane equations and line equations (on a plane and in space). Calculating distances between two points, a point and a line, a point and a plane. Calculating the distance between intersecting lines.
Calculation of angles between two lines, a line and a plane, two planes. **Topic 2. Curves and surfaces of the second order**

Curves of the second order, their geometric properties and equations. Reduction of a curve of the second order to canonical form.
Equation of a second-order surface. Canonical equations of a sphere, ellipsoid, cone, hyperboloid, paraboloid.

3.4. Topics of practical classes in the discipline "Linear Algebra"

3.4.1. Full-time education

Lesson No.	Topic of the lesson
1	**Section 1. Elements of linear algebra.**Matrices, actions over matrices.
2	Determinants, their properties and calculation. Matrix notation of a system of linear equations.
3	Inverse matrix and its calculation.
4	Systems of linear algebraic equations, basic concepts and definitions. Solution of systems of linear equations by Cramer's method, inverse matrix method
5	Solution of systems of linear equations by Gauss method. Matrix rank. Kronecker-Capelli theorem.
6	Gauss method in arbitrary systems of linear equations. Homogeneous systems of linear equations.
7	**Section 2. Elements of vector algebra.**Vectors. Linear operations on vectors, their properties. Linear space of geometric vectors. Concept of linear combination of vectors. Linearly independent and linearly dependent systems of vectors.
8	The concept of a basis for the space of geometric vectors. Vector coordinates. Orthonormal basis. Collinearity, orthogonality and coplanarity of vectors.
9	Scalar product of vectors and its properties. Vector orthogonality condition. Vector product of two vectors, its properties. Vector collinearity condition

10	Mixed product of three vectors and its properties. Calculation of volume of parallelepiped and pyramid. Condition of coplanarity of vectors. Calculation of scalar, vector and mixed products of vectors given by coordinates in orthonormal basis.
11	Linear spaces. Dimension and basis of a linear space. Matrix of transition from basis to basis. Transformation of vector coordinates when changing the basis. Linear operators, actions with linear operators. Eigenvectors and eigenvalues of a matrix
12	**Section 3. Complex numbers and polynomials**A multitude complex numbers. Forms of writing complex numbers. Operations on complex numbers. Factorization of a polynomial, the fundamental theorem of algebra
13	**Section 4. Analytical Geometry** Coordinate systems. Different types of plane equations
14	Different types of equations of a straight line on a plane and in space
15	Relative arrangement of lines on a plane and in space. Calculating distances between two points, a point and a line. Calculating the distance between intersecting lines. Relative arrangement of planes. Calculating distances between a point and a plane
16	Curves of the second order, their geometric properties. Equations of the ellipse, hyperbola, parabola. General equation of a curve of the second order. Reduction of a curve of the second order to canonical form. Surfaces of the second order

3.4.2. Part-time and part-time education

Lesson No.	Topic of the lesson
1	**Section 1. Elements of linear algebra.**Matrices, actions over matrices. Determinants, their properties and calculation.
2	Solution of systems of linear algebraic equations by Cramer's method, inverse matrix method, Gauss method.
3	**Section 2. Elements of vector algebra.**Linear opera- over vectors, their properties. Calculation of scalar, vector and mixed products of vectors. . Eigenvectors and eigenvalues of a matrix
4	**Section 3. Complex numbers and polynomials.**Actions on complex numbers. Factorization of a polynomial
5	**Section 4. Analytical Geometry**Different types of equations plane and straight lines on the plane and in space. The simplest problems of analytical geometry (calculating distances and angles)

4. Educational, methodological and informational support

4.1. Regulatory documents and GOSTs

1. Federal State Educational Standard 38.03.01. 38.03.01 Economics. Approved by the order of the Ministry of Education and Science of Russia dated 12.08.2020 No. 954.
2. Academic curriculum for the field of study: 38.03.01 Economics Profile: Economics of organizations and enterprises. Form of study – full-time. 2023.
3. Academic curriculum for the field of study: 38.03.01 Economics Profile: Economics of organizations and enterprises. Form of study – full-time and part-time. 2023.
4. Decree of the President of the Russian Federation of 01.12.2016 No. 642 “Strategies for scientific and technological development of the Russian Federation”.
5. Order of the Ministry of Education and Science of the Russian Federation dated August 23, 2017 No. 816 “On approval of the Procedure for the application by organizations carrying out educational activities of electronic learning, distance educational technologies in the implementation of educational programs.”

4.2. Main literature

1. Zabolotsky V.S. Linear algebra and analytic geometry (study complex): textbook, 2nd ed. Ster. M .: INFRA-M Publishing House, 2022. 309 p. (Higher education: bachelor's degree).
2. Sirotina I.K. Linear algebra and analytic geometry: interactive course: textbook for universities. St. Petersburg, Lan Publishing House, 2022. 180 p.
3. Lecture course on linear algebra and analytic geometry: a tutorial. // Kudryavtsev B.Yu., Matyash V.I., Pokazeev V.V., Cherkesova G.V.. Moscow: Moscow State Technical University "MAMI", 2009. - 362 p. [http://lib.mami.ru/lib/content/elektronnyy-katalog. Electronic resource.]

4.3. Further reading

1. A Brief Course in Higher Mathematics [Electronic resource]: textbook. - Electronic data. - Moscow: Dashkov i K, 2017. - 510 p. [Access mode: URL: https:// e.lanbook.com/book/93522. - Title from the screen.] .
2. Zubkov V.G., Lyakhovsky V.A., Martynenko A.I., Minostsev V.B., Pushkar E.A. Course of Mathematics for technical higher educational institutions. Parts 1-4. Moscow: MGIMO, 2012. 400 copies.
3. Ilyin V.A., Poznyak E.G. Analytical Geometry. Moscow: Nauka. Editor-in-Chief of the Physical-Mathematical Literature, 1988. 224 p.
4. Beklemishev, D.V. Solving problems from the course of analytical geometry and linear algebra [Electronic resource]: textbook - Electronic data - Moscow: Fizmatlit, 2014. - 192 p. [Access mode: URL: https://e.lanbook.com/book/59632. - Title from the screen.]

4.4. Electronic educational resources

Conducting classes and assessments is possible in a remote format using the system

University distance learning systems (LMS) based on electronic educational resources (EER) developed by the Department of Mathematics for all sections of the program:.

Name of the EOR	
Linear and vector algebra	https://online.mospolytech.ru/course/view.php?id=3368

Analytical Geometry	https://online.mospolytech.ru/local/crw/course.php?id=2843
Linear algebra and analytic geometry	https://online.mospolytech.ru/course/view.php?id=462

The developed electronic educational resources include training and final tests.
The procedure for carrying out work in a remote format is established by separate orders of the Vice-Rector for Academic Affairs and/or the Center for Educational and Methodological Work.

Each student is provided with individual unlimited access to the electronic library of Moscow Polytechnic University
https://online.mospolytech.ru/course/view.php?id=7621§ion=1 .
Internet resources include educational and methodological materials in electronic form, presented on the website mospolytech.ru in the section: "Center for Mathematical Education" (http://mospolytech.ru/index.php?id=4486 , http://mospolytech.ru/index.php?id=5822).
Useful educational, methodological and informational materials are presented on the websites: *http://exponenta.ru* , *http://eqworld.ipmnet.ru/ru/info/mathwebs.htm* .
List of resources of the information and telecommunications network "Internet" for mastering the discipline: www.matematikalegko.ru>studentu,www.i-exam.ru .

4.5. Licensed and freely distributed software security

No.	Name	Software Developer (copyright holder-body)	Availability (licensed, freely disperse distributed)	Link to the Unified Register of Russian Computer Programs and Databases (if available)
1	AstraLinuxCommonEdition	OOO "RUSBITEKH-ASTER"	Licensed	https://reestr.digital.gov.ru/reestr/305783/?sphrase_id=9540 36
2	MyOffice	OOO "NEW CLOUD TECHNOLOGIES"	Licensed	https://reestr.digital.gov.ru/reestr/301558/?sphrase_id=9433 75

4.6. Modern professional databases and information references external systems

List of Internet resources available for mastering the discipline:

	Name	Link to resource	Availability
Information and reference systems			

	StackOverflow	https://stackoverflow.com/	Available on the Internet without restrictions ny
	Information resources of the ConsultantPlus Network	http://www.consultant.ru	Available
	Portal of Federal State Educational Standards of Higher Education	http://www.fgosvo.ru.	Available
Electronic library systems			
	Doe	https://e.lanbook.com/	Available on the Internet without restrictions ny
	IPR Books	https:// www.iprbookshop.ru/	Available on the Internet without restrictions ny
Professional databases			
	Database of scientific electronic library (eLIBRARY.RU)	http://www.elibrary.ru	Available
	WebofScienceCoreCollection is a polythematic abstract-bibliographic and scientometric (bibliometric) database	http://webofscience.com	Available

5. Material and technical support of discipline

The university's material and technical base ensures the implementation of all types of classes provided for by the curriculum and complies with current sanitary and fire safety rules and regulations.

The Department of Mathematics does not have its own classrooms and uses classrooms from the general university fund.

If necessary, the university's computer labs are used to conduct interactive practical classes.

6. Methodological recommendations

6.1. Methodological recommendations for teachers on organizing training nia

First of all, it should be noted that students should be aware that almost all of the material they study does not require any special (additional) preparation and

can be successfully studied if students attend classes, complete homework assignments on time, and use (if necessary) the system of scheduled consultations during each semester. The sections included in the mathematics course are classical, at the same time they are practically oriented, since they are widely used to solve various types of problems within mathematics itself and applied problems. Their mastery will help students to logically correctly, reasonably, and clearly construct oral and written speech, and successfully apply the accumulated knowledge in their professional activities.

It is necessary to recommend basic and additional literature to students from the very beginning of classes, and at the end of the semester to provide a list of questions to prepare for the exam.

At the first lesson on a subject, it is essential to inform students about the type and form of midterm assessment on the subject, the timing of its implementation, the conditions for admission to midterm assessment, and the types of midterm assessment used.

The presentation of theoretical material should be accompanied by illustrative examples, carefully selected by the teacher so that technical difficulties and calculations in solving the problem do not distract from the main thing: understanding the idea and essence of the methods used. Examples of the practical application of the equations and formulas considered in class should always be given.

Practical classes should be organized by the teacher in such a way that there is time for students to periodically complete a short independent work in the classroom to check their assimilation of the material presented.

The teacher conducting practical classes must coordinate the educational and thematic plan of classes with the lecturer and use a unified system of notations.

The teacher should strive for systematic, continuous work of students throughout the semester; it is necessary to identify strong students and involve them in scientific work, in participation in various kinds of Olympiads and competitions.

The student must feel the teacher's interest in achieving the final result: in the acquisition by students of solid knowledge, skills and mastery of accumulated information to solve problems in professional activities.

6.1.1. Educational technologies

It is possible to conduct classes and assessments remotely using the university's distance learning system (LMS) based on electronic educational resources (EER) developed by the Department of Mathematics (see section 4.4).

The procedure for carrying out work in a remote format is established by separate orders of the Vice-Rector for Academic Affairs and/or the Center for Educational and Methodological Work.

6.2. Methodological instructions for students on mastering the discipline

Section: Elements of Linear Algebra

Matrices and determinants. First of all, the student must understand that the matrix is this is a table of numbers (and this table can have the same number of rows and columns, or it can be rectangular), and the determinant is a number written in the form of a square table, that is, determinants exist only for square matrices.

It is necessary to pay special attention to the operation of multiplication of rectangular matrices and to understand what order of matrix-product is obtained. The peculiarity of matrices also consists in the fact that the product of matrices is not permutable, that is$A.B.-B.A.$.It is mandatory to It is necessary to verify this by solving the corresponding problems.

The concept of the inverse matrix is important. It is necessary to know the condition for the existence of the inverse matrix and the algorithm for its construction. After its calculation, it is advisable to do check the correctness of the solution by performing the multiplication operation $A^{-1}A$-E(must be completed) (the identity matrix is read)

When studying determinants, it is necessary to clearly grasp the concepts of minor, algebraic complement, and know the numerous properties of the determinant. To master the technique of calculating determinants, it is advisable to choose an arbitrary determinant higher than the third order and expand it in various ways, using expansion both by rows and by columns. Pay attention to which rows (columns) are preferable to choose for expanding the determinant in order to simplify its calculation. Calculating the determinant using elementary transformations that reduce it to a triangular form is especially effective.

When studying solutions of systems of linear algebraic equations (SLAE), pay attention, first of all, to the concept of a solution of the system and the conditions for the existence of solutions depending on the relationship between the rank of the matrix, the rank of the augmented matrix of the system and the number of unknowns and equations. Pay attention to the conditions for applying Cramer's formulas and the inverse matrix method. Carefully analyze examples of solving arbitrary systems of linear algebraic equations using the Gauss method (introduction of basic and free variables).

Section: Elements of Vector Algebra

When studying this topic, pay attention to linear operations on vectors, to the concepts of linear independence and linear dependence of vectors, to the fundamental concept of a basis of a vector space (and an orthonormal basis), to the decomposition of a vector in a basis.

Know the definition, geometric properties of scalar, vector and mixed products of vectors, formulas for their calculation in vector and coordinate form. It is necessary to know and be able to check the conditions of orthogonality, collinearity and coplanarity of vectors.

Section: Complex numbers and polynomials

In this section, first of all, it is necessary to understand that a complex number is an extension of the concept of real numbers, to know the definition and three forms of writing a complex number (algebraic, trigonometric and exponential), the geometric interpretation of a complex number and the one-to-one correspondence between the set of complex numbers and the set of points of the complex plane. To know Euler's formula. Complex numbers can be represented using vectors on the complex plane. Therefore, the operations of addition and subtraction of complex numbers can be reduced to the operations of addition and subtraction of the corresponding vectors.

It is necessary to know and be able to perform the operations of multiplication, division, raising to a positive power of complex numbers written in trigonometric form, and extracting the n-th root of a complex number.

It should be noted that the set of complex numbers is closed, that is, any algebraic operation on complex numbers does not lead beyond the domain of complex numbers.

It is necessary to know the different types of factorization of a polynomial for cases when the roots of the polynomial may include multiple, complex roots. This information will be used, for example, in integral calculus when calculating integrals of fractional rational functions, when solving linear homogeneous differential equations of the nth order with constant coefficients.

Section: Analytical Geometry

When studying analytical geometry, one should first of all understand that its task is to study geometric objects using analytical methods of algebra and

mathematical analysis. This approach is based on the coordinate method, first systematically applied by R. Descartes. The basic concepts of geometry (point, line, plane) are undefined, and an axiomatic method is used to describe them, which allows one to establish a one-to-one correspondence between the set of points of a line and the set of real numbers and, thus, introduce a coordinate system. It is necessary to clearly master the various forms of the equations of a line and a plane and be able to move from one form to another. It is necessary to comprehend and successfully apply the solutions of classical problems of analytical geometry on the relative positions of lines and planes, and be able to determine angles and distances between various geometric objects.

In conclusion, we note that successful study of the discipline "Linear Algebra", acquisition of the necessary competencies, skills and abilities in mastering the mathematical apparatus requires a lot of independent work from students. Please note that the number of hours allocated to independent work in accordance with the curriculum is greater than the number of hours allocated to all types of classroom activities.

7. Evaluation Fund

The fund of assessment tools is presented in Appendix 2 to the work program and includes the following sections:

1. Methods of monitoring and assessing learning
outcomes 2. Scale and criteria for assessing learning
outcomes 3. Assessment tools
3.1. Current control
3.2. Interim assessment

Appendix 1

1. Thematic plan of the content of the discipline "Linear Algebra" in the direction of training 38.03.01 “Economy”

Profile:

Economy of enterprises and organizations

(bachelor)

Form of study – full-time

Recruitment year 2023/2024

n/n	Chapter	Semester	Week	Types of educational work, including independent students' work, and labor intensity in hours					Types of independent work students					Forms certificate-tions	
				L	P/C	Lab	SR WITH	KSR	K.R.	K.P.	RGR	Abstract	K/p	E	Z
First semester															
1.1	**Section 1. Elements of linear algebra** Matrices, operations on matrices tsami.	1	3	2	2		3								
1.2	Determinants, their properties and calculation. Matrix notation of a system of linear equations. **Issuance of tasks to RGR No. 1 on a linear basis algebra.**	1	4		2		3				+				

1.3	Inverse matrix and its calculation. Matrix notation of a system of linear equations.	1	5	2	2		3								
1.4	Systems of linear algebraic equations, basic concepts and definitions. Solution of systems of linear equations by Cramer's method, by the inverse matrix method	1	6		2		3								
1.5	Solution of systems of linear equations by the Gauss method. Matrix rank. Kronecker-Capelli theorem.	1	7	2	2		4								
1.6	Gauss method in arbitrary systems of linear equations. Homogeneous systems of linear equations. **Independent work #1 on linear algebra in the classroom**	1	8		2		4						+		
1.7	**Section 2**. Vectors. Linear operations on vectors, their properties. Linear combinations of vectors. Linearly independent and linearly dependent systems of vectors. **Issuance of tasks RGR No. 2 on vector algebra.**	1	9	2	2		4				+				
1.8	The concept of a basis. Vector coordinates. Collinearity, orthogonality and coplanarity of vectors.	1	10		2		4								

1.9	Scalar product of vectors and its properties. Vector product of two vectors, its properties.	1	11	2	2		4								
1.10	Mixed product of three vectors and its properties. Calculation of the volume of a parallelepiped. Conditions of collinearity, orthogonality and coplanarity of vectors. Orthonormal basis. Calculation of scalar, vector and mixed products of vectors given by coordinates in an orthonormal basis. **Independent work No. 2 on vector algebra in the classroom**	1	12		2		4						+		
1.11	Linear spaces. Dimension and basis of linear space. The matrix of transition from basis to basis is the transformation of vector coordinates when changing the basis. Eigenvectors and eigenvalues of a linear operator.	1	13	2	2		4								
1.12	**Section 3. Complex numbers and polynomials**The set of complex numbers. Forms of writing complex numbers. Operations on complex numbers. Factorization of a polynomial is the fundamental theorem of algebra.	1	14		2		4								
1.13	**Section 4. Analytical Geometry** Coordinate systems. Different types of plane equations	1	15	2	2		4				+				

	Issuance of tasks to RGR No. 3 for analytical Czech geometry														
1.14	Different types of equations of a straight line on a plane and in space	1	16		2		4								
1.15	Relative arrangement of lines on a plane and in space. Calculation of distances between two points, a point and a line. Relative arrangement of planes. Calculation of distances between a point and a plane **Independent work No. 3 on analytical geometry in the classroom**	1	17	2	2		4						**+**		
1.16	Curves of the second order and their geometric properties. Equations of an ellipse, hyperbola, parabola. General equation of a curve of the second order. Reduction of curves of the second order to canonical form. Surfaces of the second order	1	18		2		4								
	Certification form		**19-21**											**E**	
	Total hours per discipline			**16**	**32**		**60**				**3 RGR**		**3** myself slave.		

2. Thematic plan of the content of the discipline "Linear Algebra" in the direction of training 38.03.01 “Economy”

Profile:

Economy of enterprises and organizations

(bachelor)

Form of study – full-time and part-time

Admission year 2023/2024

n/n	Chapter	Semester	Week of Se-Mestra	Types of educational work, including independent work of students, and labor intensity in hours					Types of independent work Students					Forms certificate-tions	
				L	P/C	Lab	SRS	KSR	K.R.	K.P.	RGR	Abstract	K/p	E	Z
First semester															
1	**Section 1. Elements of linear algebra** The concept of a matrix. Types of matrices. Operations on matrices and their properties. Determinants, their properties and calculations. Calculation of determinants of different orders	1	1	2			5								
2	Determinants, their properties and calculations. Calculation of determinants of different orders. **Issuance of tasks to RGR No. 1 on a linear basis algebra**	1	2		2		5				+				
3	Inverse matrix and its calculation algorithm. Elementary transformations	1	3		2		5								

	matrices. Matrix form of recording a system of linear algebraic equations. Matrix rank														
4	SLAE, basic concepts of solution, compatibility and incompatibility of the system. Solution of systems of linear equations by Cramer's method, inverse matrix method, Gauss method. Kronecker-Capelli theorem.	1	4		2		5								
5	**Section 2. Elements of vector algebra** Linear operations on vectors, their properties. Linear dependence and independence of vectors. Concept of basis. Vector coordinates.	1	5	2			6								
6	Scalar, vector and mixed products of vectors, their properties. **Issuance of tasks RGR No. 2 on vector- algebra**	1	6		2		6				+				
7	Decomposition of a vector by a basis. Orthonormal basis. Calculation of scalar, vector and mixed products of vectors given by coordinates in an orthonormal basis. Conditions of orthogonality, collinearity, coplanarity of vectors	1	7		2		6								
8	Linear spaces. Transformation of the matrix of a linear operator when passing from basis to basis. Eigenvalues and eigenvectors of a matrix	1	8		2		6								
9	**Section 3. Complex numbers and polynomials**	1	9	2			6								

	Forms of recording, operations on complex numbers. Moivre's formula. Extracting the root of a complex number														
10	Factorization of a polynomial. Fundamental theorem of algebra **Algebra Test**	1	10		2		6						+		
11	**Section 4. Analytical Geometry** Coordinate systems. Different types of plane equations.	1	11	2			6								
12	Different types of equations of a straight line on a plane and in space **Issuance of tasks to RGR No. 3 for analytical Czech geometry**	1	12		2		6				+				
13	Relative arrangement of lines on a plane and in space. Calculating distances between two points, a point and a line. Relative arrangement of planes. Calculating distances between a point and a plane	1	13		2		6								
14	Curves of the second order, their geometric properties. Equations of the ellipse, hyperbola, parabola. General equation of a curve of the second order. Reduction of curves of the second order to canonical form. Surfaces of the second order. **Control test on analytical geometry**	1	14		2		6						+		
	Certification form		**15-18**											**E**	

	Total hours per discipline			**8**	**20**		**80**				**3 RGR**		**2** **those-** **hundred**		

Appendix 2

ASSESSMENT TOOLS FUND

BY DISCIPLINE

"Linear Algebra"

Field of study: 38.03.01 "Economics". Profile:
"Economics of enterprises and organizations".
Forms of study: full-time, part-time.

Department: "Mathematics".

1. Methods of monitoring and assessing learning outcomes

To monitor academic performance and the quality of mastering the discipline, this program provides for the following types of monitoring:

- monitoring of current academic performance (current monitoring);
- midterm assessment (exam).

2. Scale and criteria for assessing learning outcomes

Form of midterm assessment: exam

Midterm assessment of students in the form of an exam is carried out based on the results of completing all types of academic work stipulated by the curriculum for a given discipline, taking into account the results of current monitoring of academic performance during the semester. The assessment of the degree to which students have achieved the planned learning outcomes for a discipline is carried out by the teacher conducting classes in the discipline using the expert assessment method. Based on the results of midterm assessment for a discipline, a grade of “excellent,” “good,” “satisfactory,” or “unsatisfactory” is given.

Rating scale	**Description**
Great	All completed**mandatory conditions for the preparation of students denta for intermediate certification**, provided by the discipline program.The student demonstrates the appropriate the combination of knowledge, skills, and abilities with the indicators given in the tables, operates with the acquired knowledge, skills, and abilities, and applies them in situations of increased complexity. At the same time, minor errors, inaccuracies, and difficulties in transferring knowledge and skills to new, non-standard tasks may be allowed.
Fine	All completed**mandatory conditions for the preparation of students denta for intermediate certification**, provided by the program of the discipline. The student demonstrates that knowledge, skills, and abilities correspond to the indicators given in

	tables, operates with acquired knowledge, skills, and abilities. At the same time, when answering, he allows for minor errors, and solves problems with shortcomings that do not affect the overall course of the solution.
Satisfactorily	All completed**mandatory conditions for the preparation of students denta for intermediate certification**, provided by the program of the discipline. The student demonstrates the correspondence of knowledge, abilities, and skills to the indicators given in the tables, operates with the acquired knowledge, abilities, and skills. But he shows shallow knowledge, does not allow gross errors or contradictions when answering, however, in the formulation of the answer there is no proper connection between the analysis, argumentation, and conclusions, and the solution of problems may contain gross errors. To obtain the correct answer, clarifying questions are required.
Unsatisfactory	Not fulfilled**mandatory conditions for the preparation of students denta for intermediate certification**, provided by the discipline program, OR the student demonstrates incomplete compliance of knowledge, skills, and abilities with those given in the tables, significant errors are made, a lack of knowledge, skills, and abilities is demonstrated in a number of indicators, the student experiences significant difficulties in operating knowledge and skills.

3. Evaluation tools

No. p/p	Name evaluation average sva	Brief description of the evaluation means	Presentation of the assessment night remedy in FOS
1	Control (sa-independent) Job (KR)	A means of testing the ability to apply acquired knowledge to solve problems of a certain type on a topic or section	Control tasks niya (example)
2	Calculation and graphic Czech work (RGR)	A means of testing the ability to apply acquired knowledge using a predetermined methodology to solve problems or assignments for a module or discipline as a whole.	Set of tasks to perform the dis-even-graphic works

3	Oral survey conversation, (UO)	A means of control organized as a special conversation between a teaching staff member and a student on topics related to the discipline being studied, and designed to determine the extent of the student's knowledge on a specific section, topic, problem, etc.	Questions about topics/sections of the disciplins
4	Test (T)	A system of standardized tasks that allows automated implement a procedure for measuring the level of knowledge and skills of a student.	Test option
5	Examination tickets (EB)	A means of testing knowledge, skills, and abilities. May include a set of theoretical questions, tasks, and practical assignments.	Examination tickets.

3.1. Current control

Assessment tools for ongoing monitoring of academic performance include test questions and assignments in the form of form testing to monitor students' mastery of sections of the discipline, and the use of RGR.

Contents of the calculation and graphic work.

Calculation and graphic work No. 1 on linear algebra.
Brief summary of the calculation and graphic work:
Matrices and determinants. Solution of systems of linear algebraic equations by Gauss, Cramer and inverse matrix methods.
Calculation and graphic work No. 2 on vector algebra. Brief summary of the calculation and graphic work:
Vectors, operations on vectors. Scalar, vector, mixed products of vectors. **Calculation and graphic work No. 3 on analytical geometry.** Brief summary of the calculation and graphic work:
Various types of equations of a line and a plane. Mutual arrangement of lines on a plane, planes in space.

Set of tasks
to perform calculation and graphic works (CGR)
by discipline**"Linear Algebra"**

RGR No. 1, part 1

Task No. 1. Matrices are given $A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & 3 & 2 \\ -4 & 0 & -3 \end{pmatrix}$ And $B = \begin{pmatrix} -2 & 0 & 2 \\ -0 & -3 & 3 \\ -1 & 1 & -1 \end{pmatrix}$.

Perform actions: $A^2 - 8B^T$.

Task No. 2. Continue this matrix equality $(2A-3B)^2 - 4A^2 - 6A.B. - ...$

and check it for matrices A And IN from the first task.

Task No. 3. Calculate the determinant in two ways: by expansion along the first row and by expansion along the first column.

$$\begin{vmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & -2 & 1 \end{vmatrix}.$$

Task No. 4. Find the inverse matrix for the given matrix. Check.

$$\begin{pmatrix} -1 & 0 & 2 \\ -1 & 1 & 0 \\ -1 & 2 & 3 \end{pmatrix}.$$

Task No. 5. Solve the matrix equation $AXB = C$ (find X). Make a check.

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 0 & 4 \\ -9 & 3 \end{pmatrix}.$$

Task No. 6. Find the rank of the matrix.

$$\begin{pmatrix} 1 & -1 & 2 & 3 & 7 \\ 0 & 5 & 4 & 3 & 1 \\ 2 & 3 & 8 & 9 & 15 \\ 3 & -8 & 2 & 6 & 20 \end{pmatrix}$$

RGR #1, tasks for solving systems of linear algebraic equations

Task #1 . Solve the system using Cramer's method. Check.

$$\begin{cases} x_1 - 4x_2 - x_3 = -2 \\ 2x_1 - 2x_2 - 3x_3 = 7 \\ 3x_1 - 6x_2 - 2x_3 = 1 \end{cases}.$$

Task No. 2. Solve system #1 using the inverse matrix method.

Task #3 . Solve the system from No. 1 using the Gauss method.

Task #4 . Solve a non-homogeneous system using the Gauss method. Find a general solution and a particular solution.

$$\begin{cases} x_1 - x_2 - 2x_3 - 3x_4 = 5 \\ 3x_1 - 2x_2 - 4x_3 - 2x_4 = 1 \\ 5x_1 - 4x_2 - 4x_4 = 11 \end{cases}$$

Task #5 . Solve a homogeneous system using the Gauss method. Find the general solution and the FSR.

$$\begin{cases} -2x_1 - 6x_2 - 9x_3 - 9x_4 = 0 \\ -4x_1 - 5x_3 - x_4 = 0 \\ -3x_1 - 3x_2 - 7x_3 - 4x_4 = 0 \\ -x_1 - 3x_2 - 2x_3 - 5x_4 = 0 \end{cases}.$$

RGR #1, part 2

Task #1 . Show that vectors $\bar{m}, \bar{n}, \bar{p}$ form a basis in space and decompose vector $\bar{a}$ on this basis: $\bar{m} = (1, -1, 2), \bar{n} = (2, 0, -3), \bar{p} = (-2, -1, 1), \bar{a} = (5, -4, 13)$.

Task #2 . Vectors are given $\bar{m}$ And $\bar{n}$. Find out if vectors are collinear $\bar{a}$ And $\bar{b}$.

$\bar{m} = (28, -8, 8), \bar{n} = (-21, 6, -6), \bar{a} = 2\bar{m} - \bar{n}, \bar{b} = 2\bar{n} - \bar{m}$.

Task #3 . Find a, If $|\bar{m}| = 6, |\bar{n}| = 2, (\bar{m}, \bar{n}) = 135^\circ, \bar{a} = 6\bar{n} - \bar{m}$.

Task #4 . Dan - ABC. Find - B, If $A(1; -1; 2), B(3; 3; 2), C(7; 1; 2)$.

Task #5 . At what X vectors $\bar{a}$ And $\bar{b}$ perpendicular?

$\bar{a} = (x; 1; -4), \bar{b} = (x - 3; 12; x)$.

Task #6 . Find the area of a parallelogram constructed on vectors $\bar{a}$ And $\bar{b}$.

$\bar{a} = 2\bar{m} - 5\bar{n}, \bar{b} = \bar{m} - \bar{n}, |\bar{m}| = 12, |\bar{n}| = 3, (\bar{m}, \bar{n}) = \frac{\pi}{6}$.

Task #7 . Find the area of triangle ABC if A(7; 2; -3), B(6; 5; 1), C(0; -2; -7).

Task No. 8. Vectors are given $\bar{a}, \bar{b}, \bar{c}$. Find out whether the vectors are coplanar. If the vectors are not are coplanar, then find the volume of the parallelepiped constructed on the vectors $\bar{a}, \bar{b}, \bar{c}$ and define- pour - what kind of trinity they form.

$\bar{a}(1; -1; 5), \bar{b}(2; 4; -2), \bar{c}(3; 0; 1)$.

Task No. 9. For a linear operator A the images of the basis elements are given. Write $A\bar{e}_1, A\bar{e}_2, A\bar{e}_3$ the matrix of the operator A in the basis $\bar{e}_1, \bar{e}_2, \bar{e}_3$ and find the image element $\bar{x}$.

$A\bar{e}_1 = 7\bar{e}_1 - 3\bar{e}_2 - 2\bar{e}_3, A\bar{e}_2 = \bar{e}_1 - \bar{e}_2 - 4\bar{e}_3, A\bar{e}_3 = 2\bar{e}_1 - 2\bar{e}_2 - 3\bar{e}_3$,

$\bar{x} = 2\bar{e}_1 - 4\bar{e}_2 - \bar{e}_3$.

Task No. 10. Find the eigenvalues and eigenvectors of a linear operator, given in some basis by a matrix $A = \begin{pmatrix} -4 & 1 \\ -7 & 2 \end{pmatrix}$.

RGR No. 1, part 3

Task #1 . Straight l_1 passes through the point A parallel to the vector q. Straight l_2 passes through a point B perpendicular to the vector n. Find the intersection point of the lines and the angle between them, if $A(3; 5), q(1; 3), B(0; 5), n(-3; 4)$.

Task #2 . Dana is straight l_1. Straight l_2 passes through the points A And B. Find the distance from intersection points of lines l_1 And l_2 to the straight line l_3.

l_1: $x = -8t - 3$ $A(4; 0), B(7;3), l_3$: $9x - 12y - 2 = 0$.
$y = -3t - 4$,

Task #3 . Find the distance from the point M_0 to the plane - passing through the point A perpendicular to the vector n, If $M_0(0; 1; 7), A(2; -1; 4), n(6; 22; -3)$.

Task No. 4. Find the angle between the plane - and the plane - passing through the points A, B And C, If - : $5x - y - 4z - 28 = 0$, $A(5; 2; 5), B(3; 7 ; 0), C(-4; - 3; -1)$.

Task No. 5. Write down the canonical equations of a straight line given by the general equations.

$x - 2y - 9z - 10 = 0;$
$3x - 4y - 8z - 24 = 0.$

Task No. 6. Find the intersection point of the line l and planes - and the angle between them.

l: $\frac{x-2}{3} = \frac{y}{4} = \frac{z-1}{-2}$; - : $x - 2y - 3z - 12 = 0$.

Task No. 7. Straight l_1 passes through the points A And IN. Straight l_2 passes through the point M perpendicular to the plane - . Find the angle between the lines l_1 and they are in the same plane and find out if they are lying or intersect.

$A(9;- 3;1), B(4; 4; - 5), M(-1;11;-11),$ - : $2x - 3y - z - 1 = 0$.

Exercise№8. For this curve $\frac{(x-3)^2}{49} + \frac{y^2}{25} = 1$ indicate foci, eccentricity, direction tris. Construct a curve, depict foci, directrix.

Task No. 9. Give the equation of curve $9x^2 - 25y^2 - 18x - 200y - 616 = 0$ skistic to canonical view and construct a curve.

Test tasks (examples)

Option.

1. Find the values of the matrix polynomial $F(A)$

$f(x) = x^3 - 2x^2 - x - 3, A = \begin{pmatrix} -1 & 0 \\ -3 & \end{pmatrix}$

2. Find the rank of the matrix by reducing it to step form

$$\begin{pmatrix} -2 & 0 & 8 & 1 & -5 \\ -3 & -1 & 7 & 2 & 4 \\ -8 & 2 & -6 & -3 & -13 \\ -11 & -3 & 13 & 5 & 17 \end{pmatrix}$$

3. Calculate the determinant by reducing it to step form

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & -5 & 3 \\ 2 & 7 & -1 \end{vmatrix}$$

4. Find the inverse matrix of the given one (a). Solve the matrix equation (b)

(A) $\begin{pmatrix} -1 & 2 & 1 \\ 3 & -5 & 3 \\ -2 & 7 & -1 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & -2 \\ -3 & 4 \end{pmatrix} X \begin{pmatrix} -2 & 3 & 5 \\ -3 & -1 & 0 \end{pmatrix}$

5. Solve the system of equations. Indicate the general and one particular solution (a).

Solve the system using the inverse matrix and Cramer's formulas (b)

(a) $\begin{cases} -4x_1 - 9x_2 - 5x_3 - 8x_4 = 5 \\ -7x_1 - 2x_2 - 4x_3 = -4 \\ -3x_1 + 2x_2 - 5x_3 - x_4 = 3 \\ -x_1 - 2x_2 - 3x_3 - 3x_4 = 4 \end{cases}$

(b) $\begin{cases} -x_1 - 2x_2 - x_3 = 5 \\ -5x_1 - 3x_2 = -7 \\ -3x_1 + 2x_2 - 7x_3 - x_4 = -13 \\ -x_1 - 2x_2 - 3x_3 = 1 \end{cases}$

Option

1. Write down the decomposition of a vector $\vec{x}$ by vectors $\vec{p}, \vec{q}, \vec{r}$

$\vec{x}=(-5,-12,1)$, $\vec{p}=(-1,-3,0)$, $\vec{q}=(-1,-1,1)$, $\vec{r}=(0,-1,2)$

2. Are the vectors collinear? $\vec{p}$ And $\vec{q}$?

$\vec{a}=(-2,0,1)$, $\vec{b}=(-2,3,1)$, $\vec{p}=2\vec{a}-2\vec{b}$, $\vec{q}=3\vec{a}-2\vec{b}$

3. Given a pyramid with vertices at points $A_1A_2A_3A_4$.

$A_1(-2, 4,8)$, $A_2(4,-1,2)$, $A_3(-8, 7,10)$, $A_4(-3, 4, -2)$

Find:

a) angle between ribs A_1A_2, A_1A_4

b) the volume of the pyramid

c) the length of the height dropped onto the edge $A_1A_2A_3$

c) find the length of the median drawn from the vertex A_1 to the edge A_2A_3

4. Calculate the area of a parallelogram constructed from vectors $\vec{a}$ And $\vec{b}$ – angle between vectors $\vec{p}$ And $\vec{q}$

$\vec{a}=2\vec{p}-\vec{q}, \vec{b}=\vec{p}-3\vec{q}$, $|\vec{p}|=2$, $|\vec{q}|=2$, $(\vec{p},\vec{q})=\frac{\pi}{4}$

Option

1. Find the distance from the point M_0 to the plane $(M_1M_2M_3)$:

$M_0(-9;10;2), M_1(0;7;-4), M_2(4;8;-1), M_3(-2;1;3)$

2. Write out the canonical equation of a straight line: $\begin{cases} x+y+z+2=0 \\ x-y+3z+6=0 \end{cases}$

3. Find the point of intersection of the line and the plane: $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-1}{-4}$ And $x+y+2z-9=0$.

4. Find point P_1 symmetrical to the point P relatively direct l: $P(0;-1;3)$ And $\frac{x-1}{1}=\frac{y}{-1}=\frac{z}{1}$.

5. Write the equation of a plane passing through a point M_0 and perpendicular to the straight line (M_1M_2): $M_0(3; 2; 0), M_1(4;1;5), M_2(2;-1;4)$.

6. Find the cosine of the angle between the planes α_1 And α_2:

$\alpha_1: 3x+y+3=0$, $\alpha_2: x+2y+5z+10=0$

Variant of tasks for the intermediate test

1.1 Find the rank of a matrix by reducing it to step form $\begin{pmatrix} 2 & 0 & 8 & 1 & 5 \\ 3 & 1 & 7 & 2 & 4 \\ 8 & 2 & 6 & 3 & 13 \\ 11 & 3 & 13 & 5 & 17 \end{pmatrix}$

1.2. Solve the matrix equation $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} X = \begin{pmatrix} 2 & 3 & 5 \\ 3 & 1 & 0 \end{pmatrix}$

1.3. Solve the system using Cramer's formulas $\begin{cases} x_1+2x_2+x_3=5 \\ 3x_1+5x_2+3x_3=7 \\ 2x_1+7x_2+x_3=13 \end{cases}$

2.1. Write the decomposition of a vector x by vectors $\vec{p},\vec{q},\vec{r}$ $\vec{x}=(5,12,1)$, $\vec{p}=(1,3,0)$, $\vec{q}=(1,1,1)$, $\vec{r}=(0,1,2)$

2.2. Calculate the area of a parallelogram constructed from vectors a And $\vec{b}$ – angle between vectors p And $\vec{q}$

$\vec{a}=2\vec{p}-\vec{q}, \vec{b}=\vec{p}-3\vec{q}$, $|\vec{p}|=2$, $|\vec{q}|=2$, $(\vec{p},\vec{q})=\frac{\pi}{4}$

2.3. Given a pyramid with vertices at points $A_1A_2A_3A_4$. $A_1(6,1,3)$, $A_2(6, -2, -3)$, $A_3(2, 2, 0)$, $A_4(-5,1,0)$

Find the length of the median drawn from the vertex A_1 to the edge A_2A_3

3.1 Find the intersection point of the line and the plane: $\frac{x-2}{1} - \frac{y-3}{1} - \frac{z-1}{-4}$ And $x-y-2z-9 - 0$.

3.2. Find point P_1 symmetrical to the point P relatively direct $l: P-0;-1;3-$ And $\frac{x-1}{1} - \frac{y}{-1} - \frac{z.}{1}$

3.3. Write the equation of a plane passing through a point M_0 and perpendicular to the straight line $-M_1M_2- : M_0-3; 2; 0-, M_1-4;1;5- , M_2-2; -1; 4-$.

4.1. Given a second order curve $x_2- 4y_2-12x-16y-16 - 0$. Obtain the canonical equation of the curve and construct .it

4.2. Create a transition matrix from the basis e_1, e_2, e_3 to the basis e_1-, e_2-, e_3- , If

$\vec{e}_1- -2\vec{e}_1-\vec{e}_2-\vec{e}_3;\vec{e}_2- -\vec{e}_1-\vec{e}_2-\vec{e}_3;\vec{e}_3- -\vec{e}_1-2\vec{e}_2-\vec{e}_3$

Evaluation criteria:
- the grade “passed” is given to a student if he regularly submitted solutions to problems during the semester, completed all assignments in full and defended them, answering the teacher’s questions;

- the grade “fail” is given to a student if he/she has not regularly submitted solutions to problems during the semester, has not completed assignments completely or has not submitted work for checking at all, or makes significant inaccuracies in his/her answers to the teacher’s questions.

To conduct ongoing monitoring of students’ knowledge in a remote format, the online courses developed by the Department of Mathematics include intermediate (trial) tests.

3.2. Interim assessment

Intermediate assessment - (exam) is conducted orally using tickets. Time for preparing answers to questions is no more than 45 minutes.
The ticket includes a theoretical question and tasks.
Sets of examination tickets are stored in the Department of Mathematics.
Standard ticket options are attached..

Set of questions
by discipline
"Linear Algebra"

1. Matrices, types of matrices.
2. Operations with matrices, their properties.
3. Multiplication of rectangular matrices.
4. Matrix form of recording a system of linear algebraic equations.
5. Determinants and their properties.
6. The concept of a determinant. Minors and algebraic complements.
7. Systems of linear algebraic equations, basic concepts and definitions.

8. Cramer's rule for solving systems of linear equations.
9. Inverse matrix and its calculation. Condition for the existence of the inverse matrix.
10. Solution of systems of linear algebraic equations using the inverse matrix method.
11. Gauss method for solving systems of linear equations.
12. Solution of arbitrary systems of linear equations by the Gauss method.
13. Matrix rank. Kronecker–Capelli theorem.
14. Homogeneous systems of linear equations. Criterion for the existence of non-trivial solutions.
15. The concept of a vector and linear operations on vectors, properties of operations.
16. Linear combination of vectors.
17. Linear independence and linear dependence of geometric vectors. Criterion of linear dependence.
18. The concept of a basis. Vector coordinates.
19. Orthonormal basis. Decomposition of a vector into basis vectors.
20. Ordered triple of vectors.
21. Scalar, vector and mixed product of vectors in an orthonormal basis.
22. Conditions of orthogonality, collinearity and coplanarity of vectors.
23. Linear spaces.
24. Matrix of transition from basis to basis.
25. Eigenvectors and eigenvalues of a matrix.

Analytical Geometry

1. Coordinate systems. Cartesian rectangular coordinates on the plane and in space.

2. Polar coordinate system.
3. Different types of equations of a plane and a line.
4. Calculating the distance between two points, a point and a line, a point and a plane.
5. Calculating the distance between intersecting lines.
6. Calculation of angles between two lines, a line and a plane, two planes.
7. Curves of the second order, their geometric properties and equations. Reduction of a curve of the second order to a canonical form.
8. Canonical equations of an ellipse, hyperbola, parabola.
9. Equation of a second-order surface.
10. Canonical equations of a sphere, ellipsoid, cones, hyperboloids, paraboloids.

Standard ticket options

by discipline**"Linear Algebra"**

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
FEDERAL STATE AUTONOMOUS EDUCATIONAL
HIGHER EDUCATION INSTITUTION **"MOSCOW POLYTECHNIC UNIVERSITY"**
(MOSCOW POLYTECHNIC)

Faculty of Basic Competencies, Department of Mathematics
Discipline "Linear Algebra"
Course 1, Semester 1

EXAM TICKET

1. Determinants and their properties.

2. Solve the matrix equation $XB-A$, If $A=\begin{pmatrix}-1 & 2\\ -2 & 1\end{pmatrix}$, $B=\begin{pmatrix}-1 & 2\\ 0 & 1\end{pmatrix}$.

3. Solve the system using the inverse matrix method $\begin{cases}-x_1-2x_2=-1\\ -2x_1-3x_2=12\end{cases}$.

4. Vectors are given $\bar{a}=(-5,9,8)$, $\bar{b}=(-3,1,-2)$, $\bar{c}=(-8,10,-10)$. Check if they are coplanar. If not, find the volume of the parallelepiped constructed on vectors $\bar{a},\bar{b},\bar{c}$.

5. Find the angle between the vectors $\bar{a}=(1,-2,-2)$ And $\bar{b}=(2,0,1)$.

Approved at the meeting of the Department of Mathematics "03 »05 2023, protocol no.10 Acting Head of Department N.V. Vasilyeva /_____/

MINISTRY OF SCIENCE AND HIGHER EDUCATION OF THE RUSSIAN FEDERATION
FEDERAL STATE AUTONOMOUS EDUCATIONAL
HIGHER EDUCATION INSTITUTION **"MOSCOW POLYTECHNIC UNIVERSITY"**
(MOSCOW POLYTECHNIC)

Faculty of Basic Competencies, Department of Mathematics
Discipline "Linear Algebra"
Course 1, Semester 1

EXAM TICKET

1. Linear combination of vectors. Basis. Vector coordinates.

2. Calculate the determinant of the matrix $C=A^T$, If $A=\begin{pmatrix}-1 & 2 & -1\\ -3 & 1 & 0\\ -0 & 1 & 1\end{pmatrix}$.

3. Solve the system of equations using the Gauss method $\begin{cases}-4x_1-2x_2-3x_3=12,\\ x_1-5x_2-3x_3=-1,\\ -2x_1-3x_2-2x_3=3.\end{cases}$

4. Show that vectors $\bar{m}=(1,-1,2)$, $\bar{n}=(2,0,3)$, $\bar{p}=(-2,-1,1)$ form a basis in space. Write: the

5. The general equation of a straight line is given $2x-3y-5=0$. equation of a straight line with an angular coefficient; equation of a straight line in segments; normal equation of a straight line.

Approved at the meeting of the Department of Mathematics "03 »05 2023, protocol no.10 Acting Head of Department N.V. Vasilyeva /_____/

To conduct intermediate control of students' knowledge in a remote format, the online courses developed by the Department of Mathematics include final tests.